NAME:

Statistics 111 Summer Session II

Homework Four

You are allowed to discuss problems with other students, but the final answers must be your own work.

For all problems that require calculation, YOU MUST ATTACH SEPARATE PAGES, NEATLY WRITTEN, THAT SHOW YOUR WORK.

Please mark your answer in the space provided. As a general rule, each blank counts for one point unless otherwise specified. If necessary work is not shown, or if that work is substantially wrong, then you will not get credit even if the answer is correct. (The obvious purpose of this is to prevent students from mindlessly copying each other's answers.)

Report all numerical answers to at least two correct decimal places.

DUE DATE: START of class on Thursday, July 19, 2018.

1. (**Estimation**) When tossing a coin, a natural estimate of p, the probability of heads, is $\widehat{p}_1 = X/n$, where X is the number of heads. Notice that $X \sim \text{Binomial}(n, p)$. However, this means your estimate can be zero. Some people use $\widehat{p}_2 = (X+1)/(n+1)$ to avoid that.

   (a) True or False: $\widehat{p}_1$ is unbiased. Why?

   (b) True or False: $\widehat{p}_2$ is biased. Why?

   (c) What is the bias in $\widehat{p}_1$

   (d) What is the bias in $\widehat{p}_2$

   (e) What is the variance of $\widehat{p}_1$

   (f) What is the variance of $\widehat{p}_2$

2. (**Maximum Likelihood Estimators**) Let $X_1, X_2, \ldots, X_n$ be i.i.d. random sample from the distribution.

$$f(x) = \frac{1}{\theta} x^{\frac{1-\theta}{\theta}}, \quad 0 < x < 1, \quad 0 < \theta < \infty. \tag{1}$$

(a) What is the MLE of θ? Hint: your MLE should have $\ln(X)$ or something close to that as part of it. (3 points)

(b) If $Y = -\ln X$, what is the distribution of Y? Hint: you may use the change of variable method. (2 points)

(c) What is $E(Y)$? Hint: The mean of an exponentially distributed random variable $Z \sim \text{Exp}(\lambda)$ with rate parameter λ is given by $E(Z) = 1/\lambda$. Recognize the distribution of Y and use this conclusion directly! (1 points)

(d) Now show that the MLE in (a) is unbiased. Hint: (b) and (c) are also useful here. (3 points)